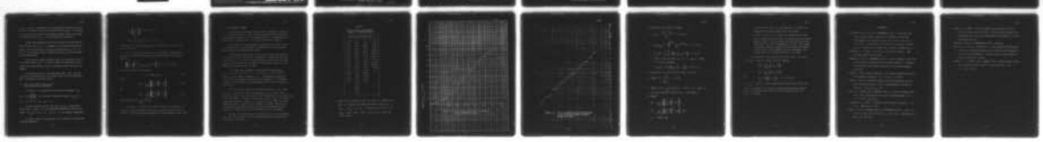
AD-A056 096 GEORGE WASHINGTON UNIV WASHINGTON D C INST FOR MANAG--ETC F/G 12/1
ESTIMATION AND TESTING OF HYPOTHESES FOR DISTRIBUTIONS IN ACCEL--ETC(U)
MAY 78 J SETHURAMAN, N D SINGPURWALLA N00014-77-C-0263

UNCLASSIFIED SERIAL-T-374 NL

| OF |

AD
A056 096



END
DATE
FILMED
8 -78
DDC

# LEVEL II

(12)
B.S.

THE
GEORGE
WASHINGTON
UNIVERSITY



STUDENTS FACULTY STUDY R
ESEARCH DEVELOPMENT FUT
URE CAREER CREATIVITY CO
MMUNITY LEADERSHIP TECH
NOLOGY FRONTIE ESIGN
ENGINEERING APP ENC
GEORGE WASHIN NIV



78 07 03

INSTITUTE FOR MANAGEMENT
SCIENCE AND ENGINEERING
SCHOOL OF ENGINEERING
AND APPLIED SCIENCE

(6)

# ESTIMATION AND TESTING OF HYPOTHESES FOR DISTRIBUTIONS IN ACCELERATED LIFE TESTS.

by

(10)

J./Sethuraman*
Nozer D./Singpurwalla**

(14) Serial-T-374

(11) 5 May 1978

(12) 23p.

(9) Scientific rept.,





The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

(15) N00014-77-C-0263, DAAG29-76-G-0238

Research Supported by

**Contract N00014-77-C-0263, Project NR 042 372
Office of Naval Research

*Grant DAAG 29-76-G0238
U.S. Army Research Office, Durham

None

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

# REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

| | | |
|---|---|---|
| 1. REPORT NUMBER<br>T-374 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle)<br>ESTIMATION AND TESTING OF HYPOTHESES FOR DISTRIBUTION IN ACCELERATED LIFE TESTS | | 5. TYPE OF REPORT & PERIOD COVERED<br>SCIENTIFIC<br>6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s)<br>J. SETHURAMAN<br>NOZER D. SINGPURWALLA | | 8. CONTRACT OR GRANT NUMBER(s)<br>N00014-77-C-0263<br>DAAG-29-76-G0238 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>THE GEORGE WASHINGTON UNIVERSITY<br>INSTITUTE FOR MANAGEMENT SCIENCE & ENGINEERING<br>WASHINGTON, D. C. 20052 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>OFFICE OF NAVAL RESEARCH, CODE 436, ARLINGTON, VA 22017<br>ARMY RESEARCH OFFICE, DURHAM. NC 27709 | | 12. REPORT DATE<br>5 May 1978<br>13. NUMBER OF PAGES<br>19 |
| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) | | 15. SECURITY CLASS. (of this report)<br>NONE<br>15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION OF THIS REPORT IS UNLIMITED.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ACCELERATED LIFE TEST
OVERSTRESS LIFE TEST
PARAMETRIC APPROACH
POWER RULE MODEL
GOODNESS OF FIT TEST
ESTIMATION OF DISTRIBUTION FUNCTIONS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper, ~~we give~~ a procedure is presented for the inference from accelerated life tests without assuming a parametric family of failure distributions at the different stress levels. Our time transformation function is a generalization of the familiar "inverse power law" relationship. We consistently estimate the failure distribution at use conditions stress, and show how to test for a specified common parametric family for the life distribution under different stresses. We illustrate our procedures by considering

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

NONE
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

NONE

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

No. 20 ABSTRACT continued.

some failure data from an industrial accelerated life test. Our results should be of interest to biometricians involved with experiments of carcinogenic substances.



NONE

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering

Abstract
of
Serial T-374
5 May 1978

# ESTIMATION AND TESTING OF HYPOTHESES FOR DISTRIBUTIONS IN ACCELERATED LIFE TESTS

by

J. Sethuraman
Nozer D. Singpurwalla

In this paper, we give a procedure for the inference from accelerated life tests without assuming a parametric family of failure distributions at the different stress levels. Our time transformation function is a generalization of the familiar "inverse power law" relationship. We consistently estimate the failure distribution at use conditions stress, and show how to test for a specified common parametric family for the life distribution under different stresses. We illustrate our procedures by considering some failure data from an industrial accelerated life test. Our results should be of interest to biometricians involved with experiments of carcinogenic substances.

Research Jointly Sponsored by
Office of Naval Research
and
U.S. Army Research Office

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Institute for Management Science and Engineering

# ESTIMATION AND TESTING OF HYPOTHESES FOR DISTRIBUTIONS IN ACCELERATED LIFE TESTS

by

J. Sethuraman
Nozer D. Singpurwalla

## 1. Introduction and Summary

Many manufactured products have a long life when used under normal conditions. It would be time-consuming to estimate the life (or failure) distribution of a product under normal or use condition stresses. One therefore tests the product under high stress conditions, makes some assumptions relating life under high stress to life under normal stress, and then estimates the life distribution under the normal stress. This, in brief, describes the problem that goes by the name of "accelerated life tests," which has also found applications in bioassay experiments.

Practitioners in this area currently make two assumptions: first, the life distributions under all stresses are assumed to belong to a common parametric family, and second, a relationship, usually referred to as a "time transformation function," is assumed among the parameters under different stresses. For example, it is common to assume that the life distributions belong to an exponential, Weibull, or lognormal family. Examples of time transformation functions are the inverse power law, the Arrhenius law, and the Eyring law [Mann, Schafer, and Singpurwalla (1974), p. 421].

In this paper we drop the assumption of a common parametric family for life distributions under all stresses but retain a version of the time

transformation function. We show how to estimate the life distribution consistently under normal stress. We then show how to test for a specified common parametric family for the life distributions under different stresses. Thus, we are able to test the validity of the first of the two assumptions made in current practice in accelerated life testing if we are willing to assume a time transformation function. In a subsequent paper [Proschan and Singpurwalla (1978)], we shall drop both these assumptions and treat the problem completely nonparametrically.

## 2. The Inverse Power Law and Its Generalized Version

The inverse power law has found applications in accelerated life testing of electronic components, and in bioassay experiments of carcinogenic substances [cf. Wagner, Berry, and Timbrell (1973)]. Levenbach (1957) has used the inverse power law for analyzing test results on paper impregnated dielectric capacitors and supported his choice of this time transformation by an empirical observation that the mean life of the capacitors is inversely proportional to an unknown power of the applied stress. Since many items, and especially capacitors, are assumed to have an exponential or a Weibull failure distribution at all stresses, the following setup for accelerated life testing has become quite conventional in engineering applications. At stress level $V_j$ , $j=1,2,\ldots,k$ , the failure distribution is assumed to be either an exponential or a Weibull, with scale parameter $\theta_j$ . At use condition $V_u$ , the failure distribution belongs to the same parametric family with scale parameter $\theta_u$ . The scale parameters are related to the stresses according to the relationship

$$\theta_j = \frac{C}{V_j^P}, \quad j=1,2,\ldots,k,u, \tag{2.1}$$

where $C$ and $P$ are unknown constants.

In bioassay experiments $V_j$ represents the dose level of a carcinogenic substance. In both accelerated life testing and bioassay

experiments it is assumed that the Weibull shape parameter is the same for all stress conditions.

In this paper we shall retain a version of the inverse power law, but do not require any assumptions about a common parametric family. Specifically, suppose that the failure distribution under the accelerated stresses $V_i$ , $i=1,2,\ldots,k$ , is $F_{V_i}$ , where $F_{V_i}$ is unknown, and that the failure distribution under use conditions stress $V_u$ is $G$ , where $G$ is also unknown. Furthermore, we assume that

$$F_{V_i}(x) = G\left(\frac{V_i^P x}{C}\right), \quad i=1,2,\ldots,k , \tag{2.2}$$

where $C$ and $P$ are unknown and $C$ depends on $V_u$ . We can verify that if $F_{V_i}$ and $G$ are exponential distributions with scale parameters $\theta_i$ and $\theta_u$ , respectively, and if Equation (2.1) holds, then Equation (2.2) follows. The same is also true if $F_{V_i}$ and $G$ are assumed to be Weibull. Thus, our model is prompted by the models previously considered in the literature.

Using failure data at the $k$ accelerated stress levels, we shall first obtain an estimator of $P$ , say $\hat{P}$ , and then obtain a consistent estimator of $G$ . Finally, we propose a method for testing the hypothesis that all the failure distributions in question belong to a common parametric family.

## 3. A Consistent Estimator of $G$

Let $X_{j,i}$ , $i=1,2,\ldots,n_j$ , denote the observed failure times under stress level $V_j$ , $j=1,2,\ldots,k$ . If the vector $(X_{j1}, X_{j2}, \ldots, X_{jn_j})$ is denoted by $\underline{X}_j$ , then from Equation (2.2)

$$\frac{V_1^P}{C}\underline{X}_1 = \frac{V_2^P}{C}\underline{X}_2 = \cdots = \frac{V_k^P}{C}\underline{X}_k \overset{\text{def}}{=} Z .$$

If $Z_{j,i} \overset{\text{def}}{=} V_j^P X_{j,i}$ , $i=1,2,\ldots,n_j$ , $j=1,2,\ldots,k$ , then the $Z_{j,i}$'s will have a common unknown distribution $G(x)$ . Equivalently, if $z_{j,i} = \log Z_{j,i}$ , $x_{j,i} = \log X_{j,i}$ , and $v_j = \log V_j$ , then $z_{j,i} = x_{j,i} + Pv_j$ , and the $z_{ij}$'s have a common distribution $H(x)$ . If $H_j(x)$ is the (unknown) distribution of the $x_{j,i}$ , $i=1,2,\ldots,n_j$ , then $H_j(x) = H(x + Pv_j)$ , $j=1,2,\ldots,k$ .

In order to estimate $P$ , we let $z_{j,i} = \mu + \varepsilon_{j,i}$ , where the $\varepsilon_{j,i}$ have mean 0 and common variance; thus $x_{j,i} + Pv_j - \mu = \varepsilon_{j,i}$ . We can now obtain the least squares estimator of $P$ , say $\hat{P}$ , using standard techniques.

Since $H_j(x - Pv_j) = H(x)$ , $j=1,2,\ldots,k$ , $\hat{H}_j(x - \hat{P}v_j)$ , the empirical distribution of $(x_{j,i} - \hat{P}v_j)$ , $i=1,2,\ldots,n_j$ , is an estimator of $H(x)$ . A pooled estimator of $H(x)$ is

$$\hat{H}(x) = \frac{\sum_{j=1}^{k} n_j \hat{H}_j(x - \hat{P}v_j)}{\sum_{j=1}^{k} n_j} . \tag{3.1}$$

The following is an immediate consequence of the Glivenko-Cantelli lemma.

Theorem 3.1: If $n_j$ is large for each $j$ , $j=1,2,\ldots,k$ , then

$$\hat{H}(x) \to H(x) \quad \text{uniformly in } x \text{ with probability } 1 .$$

Thus, we have an estimator of the distribution of the logarithms of the failure times at use conditions stress. As is well known, a

consideration of the logarithms of the failure times is not too unreasonable in life testing situations.

In the next section we shall discuss the testing of hypotheses for a specified parametric family for the distribution $H(x)$ .

## 4. Tests of Hypotheses for a Specified Common Parametric Family of Life Distributions

We shall confine our discussion to tests of hypotheses for distributions of the location-scale type. Such distributions are commonly assumed in life testing situations.

### 4.1 Preliminaries

Suppose that we wish to test the null hypothesis that the unknown distribution $H(x)$ is of the form $H_0\left(\frac{x-\alpha}{\beta}\right)$ , where $\alpha$ and $\beta$ are the unknown location and scale parameters, respectively.

Since $x_{j,i} = z_{j,i} - \beta \log V_j$ , $i=1,2,\ldots,n_j$ , $j=1,2,\ldots,k$ , testing the hypothesis that $H(x) = H_0\left(\frac{x-\alpha}{\beta}\right)$ is equivalent to testing the hypothesis that $H_j(x) = H_{0,j}\left(\frac{x-\alpha_j}{\beta_j}\right)$ . The distribution $H_{0,j}(\cdot)$ belongs to the same parametric family of distributions as $H_0(\cdot)$ , and $\alpha_j$ and $\beta_j$ are the unknown location and scale parameters, respectively, of $H_{0,j}(\cdot)$ . For example, if $H_0(x)$ is an extreme value distribution with location parameters $\alpha$ and $\beta$ , that is, if $H_0(x) = 1 - \exp\left[-\exp\left(\frac{x-\alpha}{\beta}\right)\right]$ , $-\infty < x < \infty$ , then $H_{0,j}(x)$ is also an extreme value distribution with location parameter $\alpha - \beta \log V_j$ and scale parameter $\beta$ .

We next present some results for testing the null hypothesis that $H_j(x) = H_{0,j}\left(\frac{x-\alpha_j}{\beta_j}\right)$ when $\alpha_j$ and $\beta_j$ are estimated from the data, and then adopt these for testing hypotheses about $H(x)$ .

### 4.2 Limiting distribution of a Kolmogorov-Smirnov type statistic with estimated location and scale parameters

Consider the problem of testing whether the distribution of $x_{jk}$, $x_{j2}, \ldots, x_{jn_j}$ belongs to a *specified* location and scale family, with the particular values of the location and scale parameters *not both known*. More precisely, we wish to test the null hypothesis $H_0$: $H_j(x) = H_{0,j}\left(\frac{x-\alpha_j}{\beta_j}\right)$, for all $x$ and for some values of $\alpha_j$ and $\beta_j$. Let $\left(\hat{\alpha}_{j,n_j}, \hat{\beta}_{j,n_j}\right)$ be the maximum likelihood estimators of $(\alpha_j, \beta_j)$ based upon the assumed null hypothesis distribution $H_{0,j}(\cdot)$.

We define the following quantities:

- $Y_{n_j,i} = \dfrac{x_{j,i} - \hat{\alpha}_{j,n_j}}{\hat{\beta}_{j,n_j}}, \quad 1 \leq i \leq n_j$ ;

- $W_{j,n_j}(t) = \dfrac{1}{n_j} \sum_{i=1}^{n_j} I\left[H_{0,j}\left(Y_{n_j,i}\right) \leq t\right], \quad 0 \leq t \leq 1$, where $I[E]$ denotes the indicator of the event $E$ ;

- $V_{j,n_j}(t) = (n_j)^{\frac{1}{2}}\left(W_{j,n_j}(t) - t\right), \quad 0 \leq t \leq 1$.

Let $D[0,1]$ denote the space of functions on $[0,1]$ which are right continuous and have left-hand limits, and let " $\xrightarrow{\mathcal{D}}$ " denote convergence in distribution.

Theorem 4.1 [Durbin (1973), Serfling and Wood (1976)]: Under $H_0$: $H_j(x) = H_{0,j}\left(\frac{x-\alpha_j}{\beta_j}\right)$, the "empirical stochastic process," $\{V_{j,n_j}(t);$ $0 \le t \le 1\} \xrightarrow{\mathcal{D}} V^0$ in $D[0,1]$, where $V^0$ is a Gaussian process determined by

(i) $E[V^0(t)] = 0$, $0 \le t \le 1$ and for all $0 \le s, t \le 1$;

(ii) $E[V^0(s)V^0(t)] = \min(s,t) - st$ + other terms which depend on $H_{0,j}(\cdot)$ and the properties of $\hat{\alpha}_{j,n_j}$ and $\hat{\beta}_{j,n_j}$.

A consequence of the above theorem and the continuous mapping theorem [Billingsley (1968)] is the limit law of the Kolmogorov-Smirnov test statistic

$$D_{n_j} = \max\left(D^+_{n_j}, -D^-_{n_j}\right),$$

where

$$D^+_{n_j} = \sup_{0\le t\le 1} V_{j,n_j}(t) \quad \text{and} \quad D^-_{n_j} = \inf_{0\le t\le 1} V_{j,n_j}(t).$$

The limit law of $D_{n_j}$ is given by the law of the random variable

$$D = \max(D^+, -D^-),$$

where

$$D^+ = \sup_{0\le t\le 1} V^0(t) \quad \text{and} \quad D^- = \inf_{0\le t\le 1} V^0(t).$$

Serfling and Wood (1976) and Kac, Kiefer, and Wolfowitz (1955) have obtained an expression for $E[V^0(s)V^0(t)]$ when $H_{0,j}(\cdot)$ is assumed to be a normal distribution, and when $\hat{\alpha}_{j,n_j}$ and $\hat{\beta}_{j,n_j}$ are the sample mean and the sample standard deviation, respectively. This was used to perform a Monte Carlo experiment for simulating the distribution of the limiting random variables $D^+$, $D^-$, and $D$. Serfling and Wood also

gave us a table of the quantiles of the simulated distribution that enables us to perform a test of the hypothesis that $H_{0,j}(\cdot)$ is a normal. A similar set of tables has been provided by Durbin (1975) for testing the hypothesis that $H_{0,j}(\cdot)$ is an exponential distribution.

Chandra and Singpurwalla (1977) have obtained an expression for $E[V^0(s)V^0(t)]$ when $H_{0,j}(\cdot)$ is assumed to be an extreme value distribution. This has been used to simulate the distribution of $D^+$, $D^-$, and $D$ and to obtain a table of quantiles similar to the one obtained by Serfling and Wood.

Thus, using the results described above we can perform a test of the hypothesis that $H_j(x)$ is either a normal, an exponential, or an extreme value distribution, each with estimated location and scale parameters.

Our ultimate goal is to test hypotheses about $H(x)$, the distribution of the logarithms of the failure times at the use conditions. This is discussed in the next section.

### 4.3 Tests of hypotheses about failure distribution at use conditions

In Theorem 4.1 we showed that under the null hypothesis $H_0$: $H_j(x) = H_{0,j}\left(\frac{x-\alpha_j}{\beta_j}\right)$, the "empirical" stochastic process $\{V_{j,n_j}(t)$ ; $0 \leq t \leq 1\} \xrightarrow{\mathcal{D}} V^0$, for $j=1,2,\ldots,k$.

If we now require that the life tests at the $k$ accelerated stress levels $V_1, V_2, \ldots, V_k$ are conducted independently, then the processes $\{V_{j,n_j}(t)$ ; $0 \leq t \leq 1\}$, $j=1,2,\ldots,k$ are mutually independent.

In order to state our next result, it is convenient to define the following quantities:

- $A_j = \left( \dfrac{n_j}{\sum_{j=1}^{k} n_j} \right)$, $j=1,2,\ldots,k$ ;

- $Z(t) = \sum_{j=1}^{k} A_j V_{j,n_j}(t)$ , $0 \le t \le 1$ .

We shall refer to the stochastic process $\{Z(t);\ 0 \le t \le 1\}$ as the *pooled empirical process*.

Then it follows from Theorem 3.2 of Billingsley (1968) and the properties of Gaussian processes that the pooled empirical process is such that

$$\left\{ \left( \sum_{j=1}^{k} A_j^2 \right)^{-\frac{1}{2}} \sum_{j=1}^{k} A_j V_{j,n_j}(t);\ 0 \le t \le 1 \right\} \xrightarrow{\mathcal{D}} V^0 \ , \quad \text{in } D[0,1] \ .$$

Thus the limit law of the statistic

$$D_n = \max(D_n^+, -D_n^-) \ , \tag{4.1}$$

where

$$D_n^+ = \sup_{0 \le t \le 1} \left[ \left( \sum_{j=1}^{k} A_j^2 \right)^{-\frac{1}{2}} Z(t) \right] \tag{4.2}$$

and

$$D_n^- = \inf_{0 \le t \le 1} \left[ \left( \sum_{j=1}^{k} A_j^2 \right)^{-\frac{1}{2}} Z(t) \right] , \tag{4.3}$$

is the limit law of $\sup_{0 \le t \le 1} |V^0(t)|$ .

If $H_{0,j}(\cdot)$ is chosen to be either a normal, an exponential, or an extreme value distribution, then the above result plus the results of Section 4.2 provide us with a mechanism for testing hypotheses about $H(x)$ .

## 5. An Illustrative Example

For illustrative purposes we shall apply our methods to some data on accelerated life tests. These data were abstracted from a report by Nelson (1970), and pertain to the breakdown times of an insulating fluid subjected to a voltage stress. The data are presented in Table 5.1.

Nelson assumes that the failure distribution at all stress levels is a Weibull with a scale parameter described by the power law and the shape parameter invariant with the stress.

We shall apply our methods to show that the failure distribution at all stress levels could also be a lognormal. We remark that our discussion is *purely illustrative*, and that we have no basis other than Nelson's report to believe that our model as described in Section 2.1, and particularly Equation (2.2), is valid.

If we denote the logarithms of the observed failure times by $x_{j,i}$ , $j=1,2,3,4$ and $i=1,2,\ldots,n_j$ , then the $x_{j,i}$ can be used to obtain the least squares estimator of $P$ . This estimator turned out to be 16.412; it compares very well with the estimator of $P$ obtained by Nelson, and by Al-Khayyal and Singpurwalla (1977), who also analyzed these data.

We next obtain the empirical distribution functions $H_j(x - \hat{P}v_j)$ , $j=1,2,3,4$ , and then pool these using Equation (3.1) to obtain $\hat{H}(x)$ . Recall that $H(x)$ is the distribution of the logarithms of the failure times at use conditions. In Figure 5.1 we present a plot of $\hat{H}(x)$ on ordinary graph paper, whereas in Figure 5.2 we present a plot of $\hat{H}(x)$ on normal probability paper. Since the plot in Figure 5.2 reveals linearity, we are tempted to conjecture that under the power law assumption the distribution of failure times at use conditions stress is a lognormal.

In order to confirm the above conjecture we test for the hypothesis that $H(x)$ is a normal. The necessary steps involved in executing our procedure are as follows.

TABLE 5.1

OBSERVED TIMES TO BREAKDOWN, IN MINUTES OF INSULATING FLUID

| Voltage in Kilovolts (KV) | | | |
|---|---|---|---|
| 36 | 34 | 32 | 30 |
| 1.97 | 0.96 | 0.40 | 17.05 |
| 0.59 | 4.15 | 82.85 | 22.66 |
| 2.58 | 0.19 | 9.88 | 21.02 |
| 1.69 | 0.78 | 89.29 | 175.88 |
| 2.71 | 8.01 | 215.10 | 139.07 |
| 25.50 | 31.75 | 2.75 | 144.12 |
| 0.35 | 7.35 | 0.79 | 20.46 |
| 0.99 | 6.50 | 15.93 | 43.40 |
| 3.99 | 8.27 | 3.91 | 194.90 |
| 3.67 | 33.91 | 0.27 | 47.30 |
| 2.07 | 32.52 | 0.69 | 7.74 |
| 0.96 | 3.16 | 100.58 | |
| 5.35 | 4.85 | 27.80 | |
| 2.90 | 2.78 | 13.95 | |
| 13.77 | 4.67 | 53.24 | |
| | 1.31 | | |
| | 12.06 | | |
| | 36.71 | | |
| | 72.89 | | |

1. Under the null hypothesis that each $H_j(x)$ is a normal (i.e., $H(n)$ is a normal) the maximum likelihood estimators of $\alpha_j$ and $\beta_j$, $\hat{\alpha}_{j,n_j}$ and $\hat{\beta}_{j,n_j}$, respectively, are for $j=1,2,3,$ and 4: (.902, 1.109), (1.786, 1.525), (2.228, 2.198), and (3.821, 1.111).

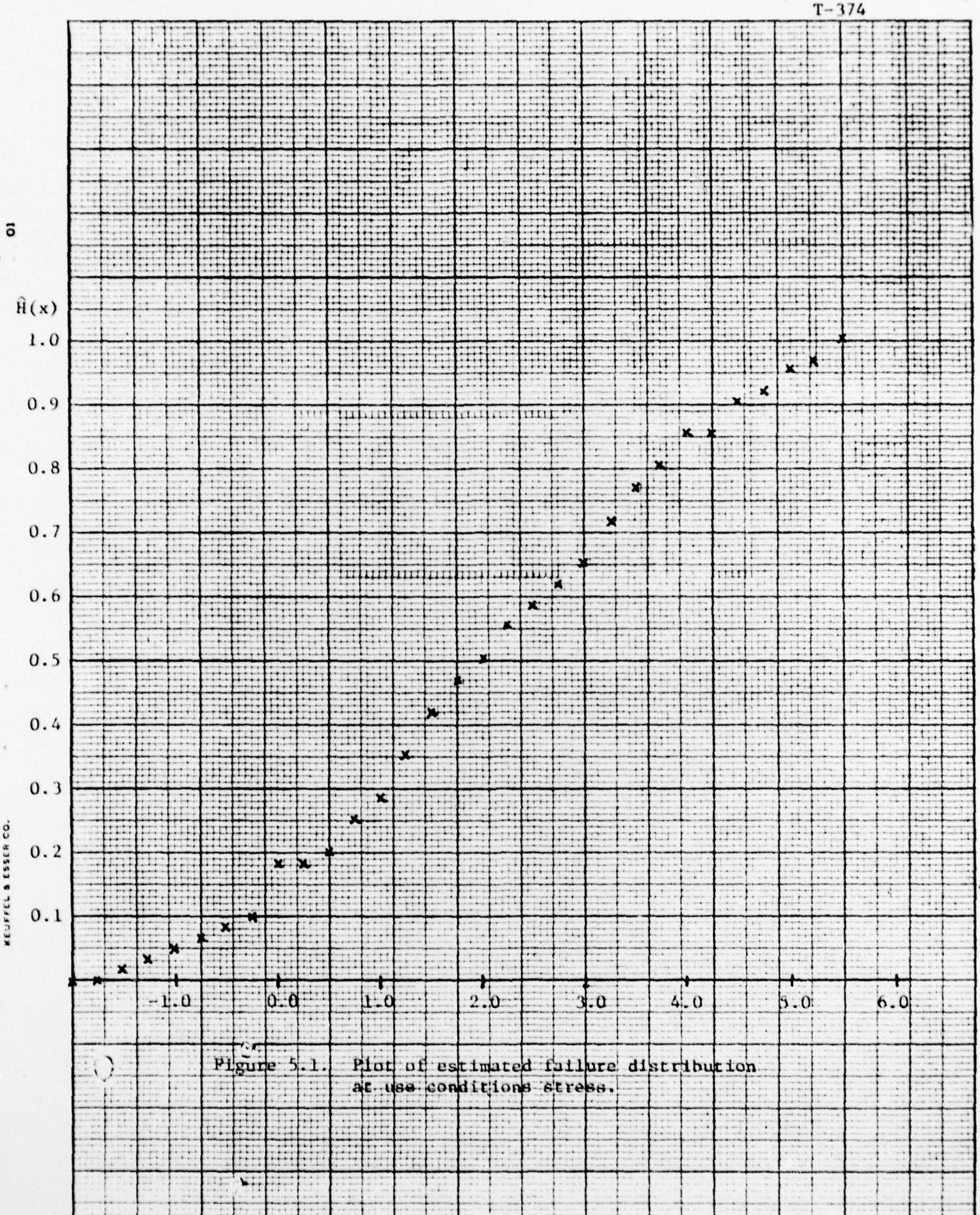



Figure 5.1. Plot of estimated failure distribution at use conditions stress.

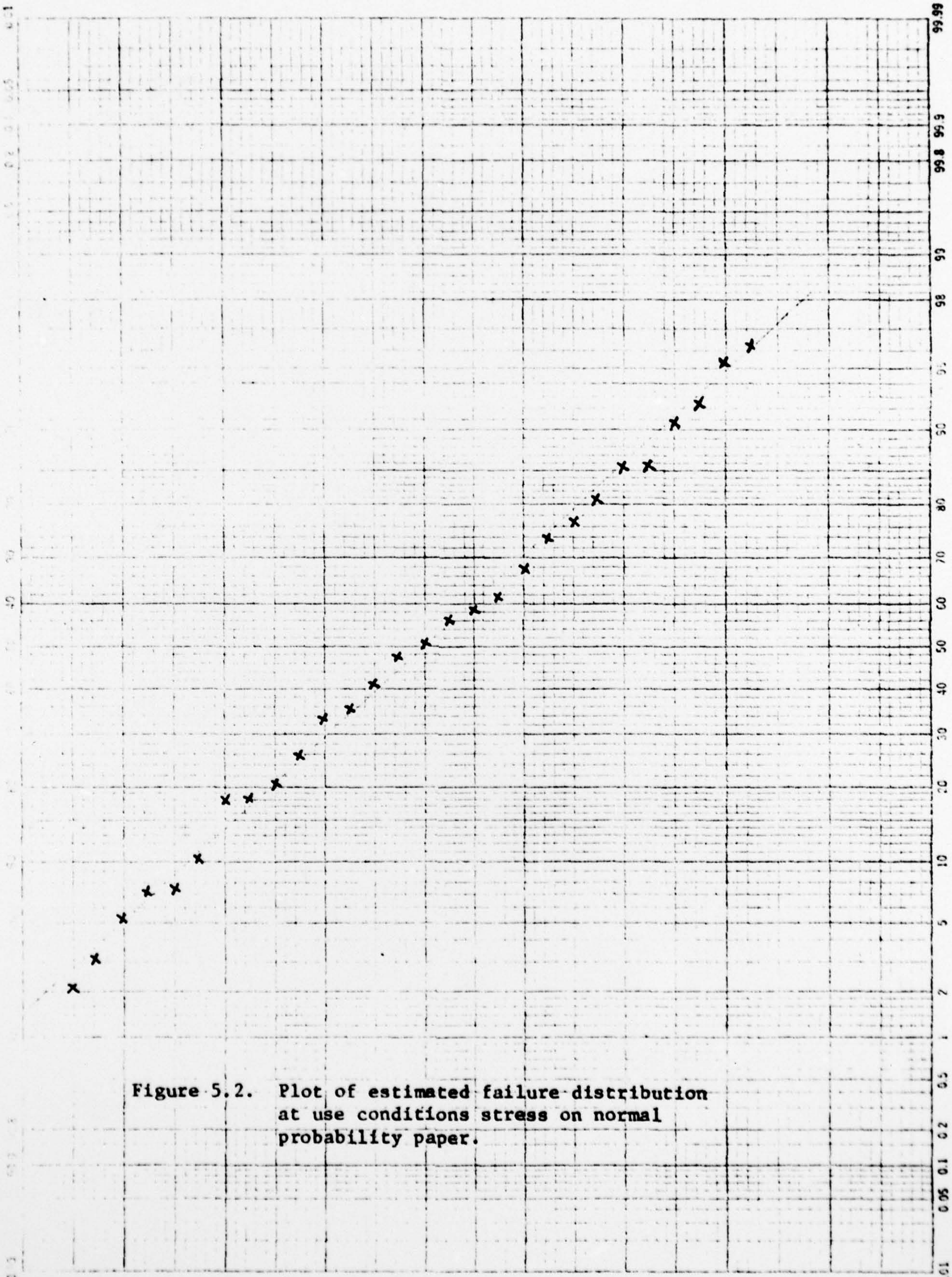



Figure 5.2. Plot of estimated failure distribution at use conditions stress on normal probability paper.

2. For the jth stress level we compute

a. $$Y_{n_j,i} = \frac{x_{j,i} - \hat{\alpha}_{j,n_j}}{\hat{\beta}_{j,n_j}} , \quad 1 \leq i \leq n_j ;$$

b. $$H_{0,j}(Y_{n_j,i}) = \int_{-\infty}^{Y_{n_j,i}} \frac{1}{\sqrt{2\pi}} e^{-S^2/2} ds , \quad 1 \leq i \leq n_j ;$$

c. $$W_{j,n_j}(t) = \frac{1}{n_j} \sum_{i=1}^{n_j} I\left(H_{0,j}(Y_{n_j,i}) \leq t\right) , \quad 0 \leq t \leq 1 ;$$

d. choose a grid of values $0 \leq t_1 < t_2 , \ldots , < t_m \leq 1$ , where $m$ is large, and obtain

$$V_{j,n_j}(t_\ell) = (n_j)^{\frac{1}{2}} \left(W_{j,n_j}(t_\ell) - t_\ell\right), \quad \ell = 1,2,\ldots,m .$$

3. Repeat Step 2 for all the stress levels $j=1,2,3,4$ .

4. Compute $A_j = \frac{n_j}{\sum_{j=1}^{4} n_j}$ , $j=1,2,3,4$ .

5. Compute $Z(t_\ell) = \sum_{j=1}^{4} A_j V_{j,n_j}(t_\ell)$ , $\ell=1,2,\ldots,m$ ; $Z(t_\ell)$ is the pooled empirical process at the $m$ grid points.

6. Compute

$$D_n^+ = \sup_{0 \leq t_\ell \leq 1} \left[\left(\sum_{j=1}^{4} A_j^2\right)^{-\frac{1}{2}} Z(t_\ell)\right] ,$$

$$D_n^- = \inf_{0 \leq t_\ell \leq 1} \left[\left(\sum_{j=1}^{4} A_j^2\right)^{-\frac{1}{2}} Z(t_\ell)\right] , \text{ and}$$

$$D_n = \max(D_n^+ , -D_n^-) .$$

In our particular case, $D_n$ was computed to be 0.4023; the computation was based choosing m=20 grid points.

7. In order to test if $D_n = 0.4023$ is significant at the 95% level of significance, we refer to the tables given by Serfling and Wood (1977). From these tables we note that the critical value at the 95% level of significance is 0.835. Thus, based on the above analysis we have no reason to reject the hypothesis that $H(x)$, the distribution of the logarithms of the failure times at use conditions stress, is a normal.

We also tested for the hypothesis that each $H_j(x)$, $j=1,2,3,4$, is a normal. This can be done easily by computing

$$D^+_{n_j} = \sup_{0 \le t_\ell \le 1} \left( V_{j,n_j}(t_\ell) \right) ,$$

$$D^-_{n_j} = \inf_{0 \le t_\ell \le 1} \left( V_{j,n_j}(t_\ell) \right) ,$$

and

$$D_{n_j} = \max\left( D^+_{n_j} , -D^-_{n_j} \right) , \quad \text{for } j=1,2,3,4 .$$

The $D_{n_j}$'s, $j=1,2,3,4$, were computed to be 0.632, 0.525, 0.434, and 0.512, respectively. Once again, by following the procedure given in Step 7, we have no reason to reject the null hypothesis that each $H_j(x)$ is a normal.

## REFERENCES

AL-KHAYYAL, FAIZ A. and N. D. SINGPURWALLA (1977). Accelerated life tests using the power law model for the Weibull distribution. Proceedings of the conference on *The Theory and Applications of Reliability: with Emphasis on Bayesian and Non-parametric Methods*, **2** (C. P. Tsokos and I. N. Shimi, eds.) 381-399. Academic Press.

BILLINGSLEY, P. (1968). *Convergence of Probability Measures*. John Wiley and Sons, Inc., New York.

CHANDRA, M. and N. D. SINGPURWALLA (1977). On some goodness of fit tests for the Weibull distribution with estimated parameters. Technical Paper Serial T-363, Program in Logistics, The George Washington University.

DURBIN, J. (1973). Weak convergence of the sample distribution function with estimated parameters. *Ann. Statist.* **1** 279-290.

DURBIN, J. (1975). Kolmogorov-Smirnov tests when parameters are estimated with applications to tests of exponentiality and tests on spacings. *Biometrika* **62** 5-22.

KAC, M., J. KIEFER, and J. WOLFOWITZ (1955). On tests of normality and other tests of goodness-of-fit based on distance methods. *Ann. Math. Statist.* **26** 189-211.

LEVENBACH, G. J. (1957). Accelerated life testing of capacitors. *IRE Trans.* PGROC No. 10, 9-20.

MANN, N. R., R. E. SCHAFER, and N. D. SINGPURWALLA (1974). *Methods for Statistical Analysis of Reliability and Life Data*. John Wiley and Sons, Inc., New York.

NELSON, W. B. (1970). Statistical methods for accelerated life test data--the inverse power law model. General Electric Research and Development Center Technical Information Series Report No. 71-C-011, Schenectady, New York.

PROSCHAN, F. and N. D. SINGPURWALLA (1978). To appear.

SERFLING, R. J. and C. L. WOOD (1976). On null hypothesis limiting distributions of Kolmogorov-Smirnov type statistics with estimated location and scale parameters. Technical report, Florida State University, Tallahassee, Florida.

WAGNER, J. C., G. BERRY, and V. TIMBRELL (1973). Mesotheliomata in rats after inoculation with asbestos and other materials. *Brit. J. Cancer* **28** 173-185.